and the Seshadri wave in the limiting cases of our theory are forward waves; however, the finite conductivity of the semiconductor makes it possible to excite backward waves. When the conductivity is increased, the discrepancy from the DE wave is first larger in smaller wavenumber and the passband becomes broader. At $N = 4$, the dispersion curve can cross over the upper bound of the frequency spectrum for the DE wave. As it approaches to the Seshadri wave, the characteristic of the backward wave disappears gradually. On the other hand, in the case of $s = +1$ where the surface wave propagates along the interface at $x = 0$, no remarkable changes of the properties can be observed [Fig. 2(b)] and this wave is similar to the DE wave.

The diagrams of $\text{Im}(F) - |\beta|$ as shown in Fig. 3 offer the information about the attenuation for the surface wave. Significant interaction between the surface wave and the electrons in the semiconductor is expected. In the absence of a bias voltage, this interaction can be regarded as the dominant cause of loss for the surface wave over the range $1 \leq N \leq 5$. Furthermore, the higher conductivity makes the semiconductor so metallic that the well-known skin effect may play an important role at $N \geq 6$. Fig. 3 implies that the optimum coupling of spins and electrons can be attained in the neighborhood of $N = 5$. In the presence of drifting carriers in the semiconductor, the wave interaction leads to the creation of the growing wave. It is natural that the loss for $s = +1$ is less than that for $s = -1$ by about -20 dB.

IV. Conclusion

The magnetostatic surface waves in the ferrite slab adjacent to the semiconductor have been investigated in the previous sections. Our analytical results have pointed out that the propagation characteristics of the surface wave are affected considerably by the finite conductivity of the semiconductor in the absence of bias voltage. In particular the backward waves are excited. It is believed that optimum coupling of spins and electrons is attained in the neighborhood of $N = 5$. If drifting carriers are present in the semiconductor, it is anticipated that the wave interaction will result in a growing wave, thereby providing gain. In addition, it may be possible to construct a voltage-tuned delay line by utilizing the composite structure constituted of YIG-film and the semiconductor.

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Coupled-Mode Analysis of Longitudinally Magnetized Ferrite Phase Shifters

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***Abstract*—Application of a coupled-mode formalism to longitudinally magnetized ferrite phase shifters provides an explanation of the increase or decrease of insertion phase with increasing magnetization which is observed in different types of phase shifters. If the higher order mode is TM, the phase shift increases with magnetization while the reverse happens if the higher order mode is TE.**

The generalized telegraphists' equations are used to analyze the TEM phase shifter. The maximum phase shift that can be obtained is determined by the effective permeability of the ferrite. However, coupling to higher order cutoff modes reduces the phase shift significantly.

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I. Introduction

In their classic paper Suhl and Walker [1] examined propagation in a ferrite-filled coaxial waveguide magnetized to saturation along the direction of propagation. They showed that, if the spacing between inner and outer conductors was sufficiently small, for the dominant (quasi-TEM) mode the ferrite could be represented as an isotropic lossless medium with an effective permeability given by

$$\mu_{\text{eff}} = \frac{\mu^2 - \kappa^2}{\mu} \tag{1a}$$

where

$$\mu = 1 + \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2} \tag{1b}$$

and

$$\kappa = \frac{\omega \omega_m}{\omega_0^2 - \omega^2} \tag{1c}$$

are the elements of the Polder tensor [2]

$$\bar{\bar{\mu}} = \mu_0 \begin{bmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

with

$$\omega_m = (2\pi\gamma)(4\pi M_s)$$
$$\omega_0 = (2\pi\gamma)H_0$$

where

- ω microwave radian frequency;
- $4\pi M_s$ ferrite saturation magnetization;
- H_0 external dc magnetic field;
- γ 2.8 MHz/Oe;
- μ_0 permeability of free space equals $4\pi \times 10^{-7}$ H/m.

Through symmetry arguments they also showed that the same result was valid for propagation in a longitudinally magnetized ferrite contained between perfectly conducting parallel planes, in the limit of small spacing between the planes.

In this paper, longitudinally magnetized ferrites in guided wave structures are examined. Coupled-mode theory is employed to gain insight into the general characteristics of such structures. A detailed analysis of the TEM phase shifter is undertaken to examine the variation of effective permeability and hence, phase shift, with plate spacing.

II. Coupled-Mode Theory

In this section we describe how differential phase shift is obtained in longitudinally magnetized ferrite structures. We approach the problem from the viewpoint of mode coupling between the dominant mode and cutoff modes capable of storing electromagnetic energy. This type of analysis has successfully predicted the behavior of the Faraday rotation phase shifter [3] and is the only plausible explanation of the Reggia–Spencer phase shifter [4]–[6].

Since the anisotropy of (2) exists only in the transverse plane, we may define a transverse tensor permeability as

$$\bar{\bar{\mu}}_t = \mu_0 \begin{bmatrix} \mu & -j\kappa \\ j\kappa & \mu \end{bmatrix}. \tag{3}$$

Now consider two transmission lines which are nonreciprocally coupled by this medium. If V_1 and I_1 represent the uncoupled voltage and current on line 1, and subscript 2 represents the same quantities on line 2, then the voltages and currents on the coupled lines satisfy the following matrix differential equations (assuming $e^{j\omega t}$ time variation) [7]:

$$\bar{\bar{V}}' = -\bar{\bar{Z}}\bar{\bar{I}} \tag{4a}$$

$$\bar{\bar{I}}' = -\bar{\bar{Y}}\bar{\bar{V}} \tag{4b}$$

where $\bar{\bar{V}}$ and $\bar{\bar{I}}$ are column vectors of voltage and current, and the prime denotes differentiation with respect to z. The series impedance matrix is

$$\bar{\bar{Z}} = \begin{bmatrix} Z_1 & -jZ_m \\ jZ_m & Z_2 \end{bmatrix} \tag{5a}$$

where the nonreciprocal coupling is represented by the off-diagonal elements $\pm jZ_m$. The shunt admittance matrix is

$$\bar{\bar{Y}} = \begin{bmatrix} Y_1 & Y_m \\ Y_m & Y_2 \end{bmatrix} \tag{5b}$$

where the capacitive coupling between the lines has been assumed to be reciprocal. The relationship between μ and Z, and κ and Z_m remains to be determined after the field expressions for the particular structure have been developed.

From (4a) and (4b) we obtain the second-order matrix differential equations

$$\bar{\bar{V}}'' - \bar{\bar{K}}^2\bar{\bar{V}} = 0$$

$$\bar{\bar{I}}'' - \bar{\bar{K}}^{2\dagger}\bar{\bar{I}} = 0$$

with $\bar{\bar{K}}^2 = \bar{\bar{Z}}\bar{\bar{Y}}$ and the dagger indicating Hermitian conjugation of the matrix. Assuming solutions of the form $e^{-\gamma z}$ yield the eigenvalue equation

$$\bar{\bar{K}}^2 - \gamma^2\bar{\bar{E}} = 0 \tag{6}$$

where $\bar{\bar{E}}$ is the unit matrix. Substitution of (5a) and (5b) into (6) yields

$$\gamma^2 = \frac{Z_1Y_1 + Z_2Y_2}{2} \pm \left[\left(\frac{Z_1Y_1 + Z_2Y_2}{2}\right)^2 - (Z_1Z_2 - Z_m^2)(Y_1Y_2 - Y_m^2)\right]^{1/2}. \tag{7}$$

From transmission-line theory we note

$$\gamma_1^2 = Z_1Y_1 \qquad \gamma_2^2 = Z_2Y_2.$$

Substitution of these into (7) and rearrangement yields

$$\gamma^2 = \frac{\gamma_1^2 + \gamma_2^2}{2} \pm \left[\left(\frac{\gamma_1^2 - \gamma_2^2}{2}\right)^2 + \gamma_1^2\gamma_2^2(k_m^2 - k_m^2k_c^2 + k_c^2)\right]^{1/2} \tag{8}$$

where

$$k_m = \frac{Z_m}{(Z_1Z_2)^{1/2}} \qquad k_c = \frac{Y_m}{(Y_1Y_2)^{1/2}}$$

are the usual definitions of magnetic and electric coupling coefficients. Note that since all quantities in (8) appear as even powers, the propagation constant is independent of the sign of either Z_m or Y_m, and reciprocal propagation is obtained. The term in (8) that produces variable phase shift is $\gamma_1^2\gamma_2^2(1 - k_c^2)k_m^2$ which may be written as $Y_1Y_2(1 - k_c^2)Z_m^2$. Since k_c^2 is constant for magnetic coupling, only the mutual impedance Z_m can vary with the state of magnetization of the ferrite.

Assuming a lossless medium, the mode admittance Y_i is imaginary and may be either positive or negative depending upon the type of mode and whether the mode is above or below cutoff. The signs of the series impedances and shunt admittances for waveguide modes are listed in Table I. If mode 1 in (8) is propagating ($\gamma_1^2 = -\beta_1^2$) and mode 2 is cutoff ($\gamma_1^2 = \alpha^2$ in the absence of coupling) then: 1) the insertion phase increases as Z_m increases if mode 2 is TM (Reggia–Spencer phase shifter); and 2) the insertion phase decreases as Z_m increases if mode 2 is TE (TEM phase shifter). The behavior predicted here has been experimentally observed for the Reggia–Spencer phase shifter [4] and for a TEM-like phase shifter [8].

TABLE I
SIGNS OF WAVEGUIDE SERIES IMPEDANCE AND SHUNT ADMITTANCE

	TEM	TE-like	TM-like
Above cutoff ($\gamma^2 < 0$)	$-jZ_i > 0$ $-jY_i > 0$	$-jZ_i > 0$ $-jY_i > 0$	$-jZ_i > 0$ $-jY_i > 0$
Below cutoff ($\gamma^2 > 0$)		$-jZ_i > 0$ $-jY_i < 0$	$-jZ_i < 0$ $-jY_i > 0$

In the next section this theory is applied in a more detailed fashion in order to obtain insight into the operation and limitations of the TEM phase shifter.

III. COUPLED-MODE THEORY OF THE TEM PHASER

For simplicity we model the TEM phaser as a parallel-plate waveguide fully filled with a longitudinally magnetized ferrite medium. The dominant mode is taken as the TEM mode of the empty waveguide. The higher order modes are taken to be the TE_{0n} modes of the parallel-plate waveguide. For this case substitution into the generalized telegraphists' equations, as derived by Schelkunoff [9], yields the coupled transmission-line equations

$$\begin{aligned}
\frac{dV_0}{dz} &= -j\omega\mu\mu_0 I_0 - \omega\mu_0\kappa \sum_{n=1}^{\infty} I_n \iint \frac{\partial T_0}{\partial y}\frac{\partial T_n}{\partial y}\,ds \\
\frac{dI_0}{dz} &= -j\omega\epsilon V_0 \\
\frac{dV_m}{dz} &= \omega\mu_0\kappa I_0 \iint \frac{\partial T_0}{\partial y}\frac{\partial T_m}{\partial y}\,ds - j\omega\mu_0\mu \sum_{n=1}^{\infty} I_n \iint \frac{\partial T_n}{\partial y}\frac{\partial T_m}{\partial y}\,ds \\
\frac{dI_m}{dz} &= -j\omega\epsilon \sum_{n=1}^{\infty} V_n \iint \frac{\partial T_n}{\partial y}\frac{\partial T_m}{\partial y}\,ds - \frac{\chi_m^2}{j\omega\mu_0} V_m.
\end{aligned} \tag{9}$$

Here V_0 and I_0 are the equivalent voltage and current of the TEM mode, and V_m and I_m are equivalent voltages and currents which represent the transverse fields of the TE_{0n} modes. The mode functions and cutoff wavenumbers are defined by

$$\begin{aligned}
T_0 &= \frac{1}{(ab)^{1/2}}\, y \\
T_m &= \left(\frac{2b}{a}\right)^{1/2} \frac{1}{m\pi} \cos\frac{m\pi}{b}\, y \\
\chi_m &= \frac{m\pi}{b}
\end{aligned} \tag{10}$$

where b is the plate separation and a is a unit width in the x direction. Since the parallel-plate waveguide is infinite in the x direction, no variation with x is allowed in the mode functions. The integrals in (9) are taken over the rectangular cross section ($a \times b$). Evaluation of these integrals leads to the series-impedance and shunt-admittance matrices:

$$\bar{\bar{Z}} = j\omega\mu_0 \begin{bmatrix}
\mu & -j\kappa\frac{8^{1/2}}{\pi} & -j\kappa\frac{8^{1/2}}{3\pi} & -j\kappa\frac{8^{1/2}}{5\pi} & \cdots \\
j\kappa\frac{8^{1/2}}{\pi} & \mu & 0 & 0 & \cdots \\
j\kappa\frac{8^{1/2}}{3\pi} & 0 & \mu & 0 & \cdots \\
j\kappa\frac{8^{1/2}}{5\pi} & 0 & 0 & \mu & 0\cdots \\
\cdot & \cdot & \cdot & 0 & \cdots \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot
\end{bmatrix} \tag{11}$$

$$\overline{\overline{Y}} = j\omega\epsilon \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 1-\frac{\chi_1^2}{k_1^2} & 0 & 0 & \cdots \\ 0 & 0 & 1-\frac{\chi_3^2}{k^2} & 0 & \cdots \\ 0 & 0 & 0 & 1-\frac{\chi_5^2}{k^2} & 0\cdots \\ \cdot & \cdot & \cdot & 0 & \cdots \\ \cdot & \cdot & \cdot & \vdots & \vdots \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \tag{12}$$

where $\epsilon = \epsilon_0\epsilon_f$ is the permittivity of the ferrite and $k^2 = \omega^2\mu_0\epsilon$. Forming the $\overline{\overline{K}}^2$ matrix and substituting into (6) gives

$$\overline{\overline{K}}^2 - \beta^2\overline{\overline{E}}$$

$$= \begin{bmatrix} \mu - \frac{\beta^2}{k^2} & -j\kappa\frac{8^{1/2}}{\pi}\left(1-\frac{\chi_1^2}{k^2}\right) & -j\kappa\frac{8^{1/2}}{3\pi}\left(1-\frac{\chi_3^2}{k^2}\right) & \cdots \\ j\kappa\frac{8^{1/2}}{\pi} & \mu\left(1-\frac{\chi_1^2}{k^2}\right)-\frac{\beta^2}{k^2} & 0 & \cdots \\ j\kappa\frac{8^{1/2}}{3\pi} & 0 & \mu\left(1-\frac{\chi_3^2}{k^2}\right)-\frac{\beta^2}{k^2} & 0\cdots \\ j\kappa\frac{8^{1/2}}{5\pi} & 0 & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}. \tag{13}$$

Expansion of (13) yields the following equation for the eigenvalues:

$$\left(\mu - \frac{\beta^2}{k^2}\right) - a_{12}^2 \sum_{n=1,3,5,\cdots}^{\infty} \frac{r_n}{n^2(\mu r_n - \beta^2/k^2)} = 0 \tag{14}$$

where

$$a_{12} = \frac{8^{1/2}}{\pi}\kappa, \qquad r_n = 1 - \frac{\chi_n^2}{k^2}.$$

If only the TE_{01} mode is coupled to the TEM mode, this reduces to the biquadratic equation

$$\left(\frac{\beta}{k}\right)^4 - \left(\frac{\beta}{k}\right)^2 [\mu(1+r_1)] + r_1(\mu^2 - a_{12}^2) = 0. \tag{15}$$

A polynomial equation of order n in $(\beta/k)^2$ results when $n-1$ higher order modes are coupled to the TEM mode. The largest positive value of β^2 obtained from the solution to (14) or (15) is the propagation constant of the dominant coupled mode.

By changing the magnetization of the ferrite, hence μ and κ, variable phase shift is obtained. We define the normalized differential phase shift as

$$\frac{\Delta\beta}{\beta_0} = \frac{\beta - \beta_0}{\beta_0} = \frac{\beta}{\beta_0} - 1 \tag{16}$$

where β is the propagation constant in the magnetized state and β_0 is the propagation constant in the demagnetized state.

When the applied field is zero, as is the case for latched operation, the Polder tensor is no longer valid. For the partially magnetized state Schlömann has recently shown [10]

$$\mu_d = \frac{1}{3} + \frac{2}{3}\left[1 - \left(\frac{\omega_m}{\omega}\right)^2\right]^{1/2} \tag{17}$$

$$\mu = \mu_d + (1-\mu_d)\left(\frac{M_r}{M_s}\cdot\frac{M}{M_r}\right)^{3/2}$$

$$\kappa = \frac{\omega_m}{\omega}\left(\frac{M}{M_s}\right) \tag{18}$$

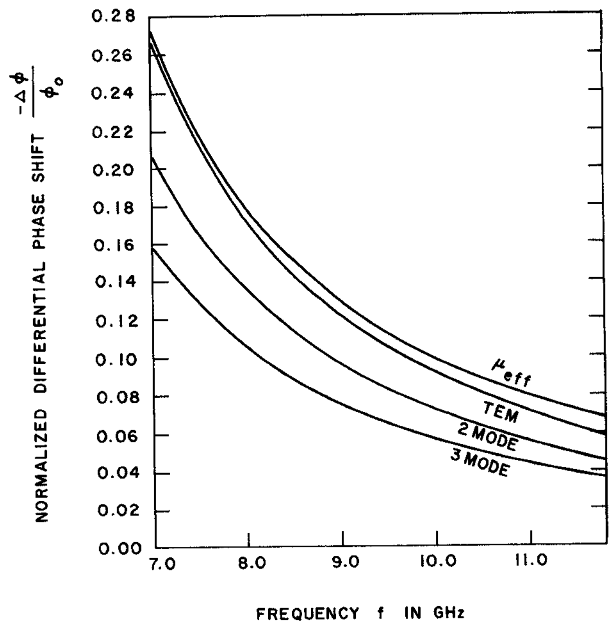

Fig. 1. Normalized differential phase shift as a function of frequency comparing several degrees of approximation with the μ_{eff} limit.

where

M_s saturation magnetization;
M_r remanent magnetization;
M magnetization.

The propagation constant in the demagnetized state is given by

$$\beta_0 = k(\mu_d)^{1/2}.$$

The differential phase shift of (16) is then

$$\frac{\Delta\phi}{\phi_0} = \frac{(\Delta\beta)l}{\beta_0 l} = \frac{\beta}{k(\mu_d)^{1/2}} - 1 \tag{19}$$

where l is the length of the device.

The negative of (19) is plotted in Fig. 1 for a 0.050-in plate spacing. Here we show the 2-mode and the 3-mode approximations as well as the μ_{eff} limit determined from (1a). In the limit of low frequencies (far from the cutoffs of the higher order modes) the approximations should approach the μ_{eff} limit since the assumption of small plate spacing is better satisfied here. Apparently, it is necessary to couple even more modes to more nearly represent the true field distribution in the parallel-plate waveguide. Notice in (11) the presence of the factor $8^{1/2}/\pi$ in all the mutual series impedances. This term results from the integration of the product of the linear mode function of the TEM mode and the sinusoidal mode function of the TE_0 modes. As shown in the Appendix a more efficient quadratic approximation is given by (A5). The results of this approximation are shown in Fig. 1 and denoted as the TEM approximation. For very thin spacings ($\leq$0.025 in) the results of (A5) are indistinguishable from the μ_{eff} limit at low frequencies.

The TEM approximation, (A5) was used to investigate the effects of plate spacing on differential phase shift. The results are shown in Fig. 2. From this we see that, as the cutoff frequency is reduced (plate spacing is increased), the normalized differential phase shift deviates drastically from the μ_{eff} limit and for some cases goes through zero and takes on negative values. Thus a given phaser configuration can exhibit either positive or negative differential phase shift depending upon the frequency of operation. The occurrence of positive and negative phase shift in TEM phasers depending upon plate spacing and frequency has been noted by Brodwin [11] and Buck [12]. The frequency of the zero crossing is dependent upon the height of the waveguide and the magnetization of the ferrite.

IV. Conclusion

The effect of a nonreciprocal gyromagnetic medium on the coupling between two guided wave modes, one propagating and one

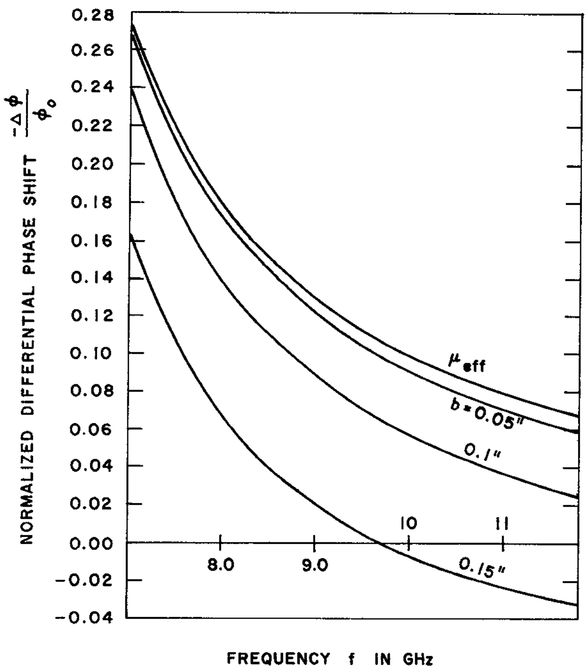

Fig. 2. Normalized differential phase shift as a function of frequency with plate spacing as the parameter.

cutoff, is to produce variable insertion phase which changes with the mutual series-impedance term. The insertion phase increases if the cutoff mode is TM-like and decreases if the cutoff mode is TE-like. This points out the fundamental difference between the Reggia–Spencer and the TEM phase shifters.

The TEM phase shifter was analyzed by applying the generalized telegraphists' equations in order to evaluate the effects of coupling to many higher order modes. A quadratic approximation was found which demonstrates the reduction in phase shift from the μ_{eff} limit, which is caused by coupling to higher order modes. When $kb \approx 1$ this reduction is negligible. The frequency dependence of the differential phase shift will always be greater than that given by the μ_{eff} limit. Furthermore, positive and negative phase shifts can occur in different frequency ranges for the TEM device depending upon plate spacing.

Appendix

The polynomial equation for the eigenvalues is given by (14) and is repeated here for convenience

$$\left(\mu - \frac{\beta^2}{k^2}\right) - a_{12}^2 \sum_{n=1,3,5,\cdots}^{\infty} \frac{r_n}{n^2(\mu r_n - \beta^2/k^2)} = 0. \quad \text{(A1)}$$

When the plate spacing approaches zero, the r_n approach infinity and (A1) reduces to

$$\left(\mu - \frac{\beta^2}{k^2}\right) - \frac{8}{\pi^2}\frac{\kappa^2}{\mu} \sum_{n=1,3,5,\cdots}^{\infty} \frac{1}{n^2} = 0. \quad \text{(A2)}$$

Solution of this yields

$$\frac{\beta^2}{k^2} = \frac{\mu^2 - \kappa^2}{\mu} = \mu_{eff} \quad \text{(A3)}$$

which is the expression derived by Suhl and Walker [1].

A better two-mode approximation than (15) in the text is obtained by rewriting (A1) as

$$\left(\mu - \frac{\beta^2}{k^2}\right) - a_{12}^2 \frac{r_1}{\mu r_1 - \beta^2/k^2} - a_{12}^2 \sum_{n=3,5,7,\cdots}^{\infty} \frac{r_n}{n^2(\mu r_n - \beta^2/k^2)} = 0. \quad \text{(A4)}$$

If only the dominant mode is of interest, $\beta/k < 1$ and the last term of (A4) may be written

$$a_{12}^2 \sum_{n=3,5,7,\cdots}^{\infty} \frac{r_n}{n^2(\mu r_n - \beta^2/k^2)} \approx a_{12}^2 \sum_{n=3,5,7,\cdots}^{\infty} \frac{1}{\mu n^2} = \frac{a_{12}^2}{\mu}\left(\frac{\pi^2}{8} - 1\right).$$

Substituting this result into (A4) yields the biquadratic equation

$$\left(\frac{\beta^2}{k^2}\right)^2 - \frac{\beta^2}{k^2}\left[\frac{\mu^2 - \kappa^2}{\mu} + \mu r_1 + \frac{a_{12}^2}{\mu}\right] + \mu r_1\left(\frac{\mu^2 - \kappa^2}{\mu}\right) = 0. \quad \text{(A5)}$$

which is the desired result.

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Experimental and Computed Four Scattering and Four Noise Parameters of GaAs FET's Up to 4 GHz

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Abstract—**The four scattering parameters, operating in the pinch-off mode, of a Schottky-barrier-gate FET (MESFET) are investigated with the aid of an appropriate equivalent circuit. The dependence of the electron drift velocity on the electric field of the channel has been simplified to be piecewise linear by Turner and Wilson. Hot electron effects have therefore been neglected. The four noise parameters of the device have also been computed using the noise sources of van der Ziel. All computed parameters are compared with their measured values in the frequency region 0.5–4 GHz. Investigated GaAs FET's are commercial units.**

I. Introduction

An appropriate equivalent circuit of a GaAs FET valid up to 4 GHz is presented here. The channel of this transistor is n-doped, with a carrier concentration of about 3×10^{16} cm^{-3}. The gate length of the device is about 4 μm and the channel width is about 360 μm. The GaAs FET is mounted in a microstrip package with three terminals. The equivalent circuit of such an FET valid up to 4 GHz has been computed using a computer-aided optimization program, based on the classical gradient method. The established network equations have been analyzed using the SYMBAL computer language [8]. Theoretical work on noise in FET's has been described by van der Ziel [2], [3] and Leupp and Strutt [9]. These data have been applied to the intrinsic GaAs FET without any modification.

The computations on the small-signal behavior as well as on the noise behavior, i.e., the four scattering parameters and the four noise parameters, have been made neglecting hot electron effects [15],

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